

I-Love irrotationally

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In this short note, we investigate the existence of universal relations between the gravimagnetic Love number of irrotational stars and the dimensionless moment of inertia. These Love numbers take into account the internal motion of the fluid, while the star is globally irrotational. The goal is to extend the so-called I-Love-Q relations - providing a strong correlation between the gravitoelectric Love number, the dimensionless moment of inertia and the dimensionless rotation-induced quadrupole - to the gravitomagnetic sector, where internal motion is taken into account. As a byproduct, we present for the first time this new gravitomagnetic Love number for realistic equations of state.

I. INTRODUCTION

Love numbers encode the deformation of self-gravitating fluids towards external tidal perturbations. While tidal interactions have been well understood for more than a century in the Newtonian context, [1], their relativistic counterpart has only started being understood recently. First studies investigated the adiabatic limit of static configurations [2, 3], i.e. under the assumption that the external tidal field varies slowly. Recently, the formalism has been extended to slowly rotating configurations [4, 5], still in the adiabatic limit. A formalism to extend the tidal response beyond the adiabatic limit, relying on effective field theory has been introduced in the Newtonian limit in [6] and extended to the relativistic case in [7].

In the relativistic case, static configurations are affected by two sector of gravitational perturbations: odd and even, corresponding to gravitomagnetic and gravitoelectric perturbations. The gravitomagnetic sector is particularly relevant to the case where rotation is present. This is a purely relativistic effect and doesn't have a Newtonian analogue: a central body surrounded by an orbiting companion will eventually be spun up, or encounter internal motion, precisely because of the gravitomagnetic field generated by the companion body.

Gravitomagnetic tidal interactions were first investigated in the post-Newtonian framework in [8], and in the relativistic regime in [2, 3]. However, it has been recently shown that taking into account the internal motion of the fluid driven by the gravitomagnetic perturbation leads to dramatically different values than the gravitomagnetic Love numbers assuming a strict hydrostatic equilibrium [9]. It was argued in [9] that this assumption of strict equilibrium, i.e. no internal motion, is too restrictive for realistic situations.

The value of the tidal coefficients depends on the equation of state of the fluid. However, it was shown in [10] that gravitoelectric Love number of neutron stars

enjoy universal properties, i.e. they are strongly correlated with the moment of inertia and the rotation induced quadrupole. These universal relations, dubbed I-Love-Q relations involve dimensionless Love number, moment of inertia and quadrupole. This result drew a lot of attention, in particular, the universality of the rotation induced multipoles have been extended to higher multipoles [11–13] leading to an approximate no hair conjecture for neutron stars. The validity of the universal relations has been extended to the case of rapid rotation in [14, 15], for frequency corrected Love numbers [16] and magnetized neutron stars [17]. These universal relations have further been extended some class of alternative models of gravity [10, 18–21], with the conclusion that they remain undistinguishable from predicted by general relativity, in the regime already constrained by observation, with the exception of the dynamical Chern Simons theory [22], though its viability can be questioned in the non-perturbative regime [23]. These universal relations have been recently used for incorporating tidal effects in waveform simulations in the post-Newtonian framework in [24]. Importantly, universal relations between various types of Love numbers (shape, electric and magnetic) have been investigated in [25]. In particular, it was found that the quadrupole gravitoelectric and gravitomagnetic Love numbers (with strict hydrostatic equilibrium) are correlated with deviation of the order of 10% for realistic equations of state.

In this paper, we reconsider the universal properties of the quadrupolar gravitomagnetic Love number, under the assumption of internal motion in the star, along the line of [9]. In Sec. II, we review the model and equations for describing slowly rotating neutron stars and gravitomagnetic perturbation of neutron stars. In Sec. III, we present our results for the gravitomagnetic Love number universality. In particular, we show that a) the gravitomagnetic Love numbers of irrotational bodies are correlated with the rescaled moment of inertia and b) that the universality is enhanced for irrotational bodies, in comparison to strict hydrostatic equilibrium. We summarize our results in Sec. IV.

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II. SLOWLY ROTATING STARS AND GRAVITOMAGNETIC LOVE NUMBER

We use the following line element

$$ds^2 = -b(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) + \epsilon_r(\omega(r) - \omega_\infty)dt + \epsilon_m(h_t(r)dt + h_r(r)dr)Y_i dx^i, \quad (1)$$

where ϵ_r and ϵ_m are respectively the small expansion parameters for slow rotation and tidal deformation, ω_∞ is the pulsar frequency measured by a distant observer, Y_i is the odd parity vector harmonic, given by $Y_i = (-\frac{1}{\sin\theta}\partial_\varphi, \sin\theta\partial_\theta)Y_{lm}(\theta, \varphi)$, where Y_{lm} is the standard scalar spherical harmonic, and $x^i = (\theta, \varphi)$.

We model the neutron star fluid by a perfect fluid

$$T_{ab} = (\rho + P)u_a u_b + P g_{ab}, \quad (2)$$

where g is the metric, ρ is the energy density, P is the pressure and u is the four velocity, given by

$$u^a = \frac{1}{\sqrt{b}}(1, 0, 0, \epsilon_r(\omega - \omega_\infty))^a \quad (3)$$

up to order ϵ_r^1 and ϵ_m^0 .

We expand the Einstein equations to first order in ϵ_r with $\epsilon_m = 0$ for slowly rotating neutron stars, and to first order in ϵ_m with $\epsilon_r = 0$ for gravitomagnetic tidal deformations.

The slow rotation equations are given by [26]:

$$M' = 4\pi G r^2 \rho, \quad b' = \frac{2b(4\pi G r^3 P + M)}{r(r - 2M)}, \quad P' = -(P + \rho)\frac{b'}{2b},$$

$$\omega'' = \frac{4\omega'(\pi G r^2(P + \rho) - f)}{r f} + \frac{16\pi G \omega(P + \rho)}{f}. \quad (4)$$

We follow [9] for computing the gravitomagnetic perturbations equations leading to

$$r^2 h_t'' + \mathcal{P} h_t' + V(r) h_t = 0, \quad (5)$$

where

$$\mathcal{P} = -\frac{4\pi r^3(P(r) + \rho(r))}{f(r)}, \quad (6)$$

$$V = \frac{-2f(r) + 8\pi \eta r^2(P(r) + \rho(r)) - (l+2)(l-1)}{f(r)},$$

where $\eta = -1$ in [27, 28]. Recently, Landry and Poisson have shown that $\eta = 1$ corresponds to the perturbation of a non rotating fluid with internal motion [9], named irrotational fluid. Here we consider both cases.

In the following, we will work in geometric units, $G = c = 1$. The vacuum solution to the slow rotation equations is given by

$$b = f = 1 - \frac{2M_s}{r}, \quad \omega = \omega_\infty \left(1 - \frac{2I}{r^3}\right), \quad (7)$$

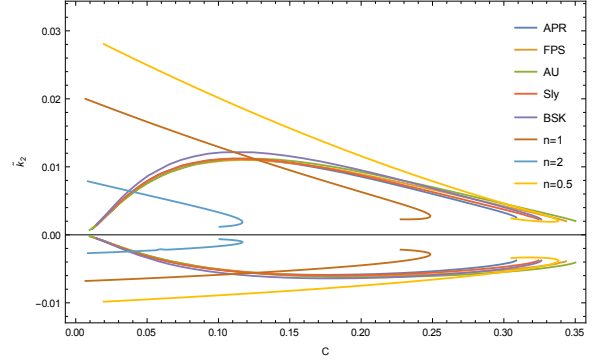


FIG. 1. The gravitomagnetic Love numbers for irrotational stars (negative) and strict equilibrium (positive), for realistic tabulated equations of state, and three polytropes, as a function of the compactness.

where I is the moment of inertia and M_s the mass of the star.

The vacuum solution of gravitomagnetic perturbation equation is expressed in terms of hypergeometric functions:

$$h_t = \frac{2}{3(l-1)} r^{l+1} \left[A_4 - 2 \frac{(l+1)}{l} K_l^{mag} \frac{2M^{2l+1}}{r} B_4 \right] \quad (8)$$

where

$$A_4 = {}_2F_1 \left(-l+1, -l-2; -2l; \frac{2M}{r} \right),$$

$$B_4 = {}_2F_1 \left(l-1, l+2; 2l+2; \frac{2M}{r} \right). \quad (9)$$

We define the following dimensionless quantities

$$C = \frac{M_s}{R_s}, \quad \bar{I} = \frac{I}{M_s^3}, \quad \tilde{k}_l = K_l (2C)^{2l}, \quad (10)$$

where C is the compactness, and R_s is the radius of the star. Note that [9] defines the compactness as $2M_s/R_s$.

Our goal is to devise a correlation between \bar{I} and the gravitomagnetic Love number, in particular, we will focus on the case $l = 2$. We use tabulated equations of state for describing the interior of the Neutron Star.

Before doing so, we present for the first time the gravitomagnetic Love numbers of irrotational stars with realistic equations of state in Fig. 1, together with the Love numbers of strict hydrostatic equilibrium. We note that the gravitomagnetic numbers of irrotational bodies tend to 0 for realistic equations of state. For completeness, we show the gravitomagnetic Love numbers for the polytropic equation of state used in [9] for $n = 0.5, 1, 2$; our result agrees perfectly with those of Fig. 1 in [9].

III. GRAVITOMAGNETIC LOVE NUMBER UNIVERSALITY

We have integrated the model equations for 5 representative equations of state, namely APR [29], BSK14 [30],

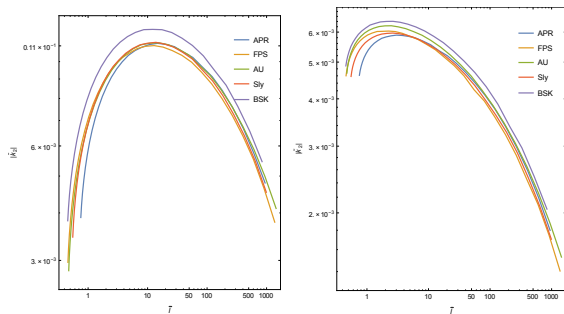


FIG. 2. The gravitomagnetic Love number for stars in strict hydrostatic equilibrium (left) and for irrotational stars (right) as a function of the dimensionless moment of inertia (right) in a Log-Log plot, for various equations of state.

AU (called AV14+UVII in [31]), SLy [32] and FPS [33], for a range of neutron star mass and radius. We show the dependence of the gravitomagnetic Love number for strict hydrostatic equilibrium and for irrotational stars and stars in strict hydrostatic equilibrium in Fig. 2 as a function of the dimensionless moment of inertia.

Our results indicates that both gravitomagnetic Love number, for the strict equilibrium and for irrotational stars enjoy a quasi universal relations. We find the largest deviation between the best fit and our data for irrotational stars (resp. stars in strict equilibrium) is of the order of 5% (resp. 10%), and most of the equations of state are within $\approx 2\%$ (resp. $\approx 5\%$). It is in principle possible to reduce this error, along the lines of [34]. We did not find a significant enhancement, but we could reduce the maximum deviation by almost a factor 2 in the case of strict hydrodynamic equilibrium. Note that the maximal deviation takes into account less compact stars where it is expected that universality deteriorates. In all case, we found universality with less than $\approx 5\%$ for most of the range of compactness that we have investigated. We use the following form of the function to fit the relations

$$\log(C^b \bar{k}_2) = d_0 + d_1 \log(C^a \bar{I}) + d_2 \log(C^a \bar{I})^2 + d_3 \log(C^a \bar{I})^3 + d_4 \log(C^a \bar{I})^4, \quad (11)$$

The coefficients are given in Table II for irrotational stars and Table I for stars in strict hydrostatic equilibrium.

Note that the deviation is slightly lower for the irrotational case than for the strict hydrostatic equilibrium, which is pleasant feature since this case is supposed to be more realistic.

We illustrate the deviations for the irrotational case in Fig. 3 and for the strict hydrostatic equilibrium in Fig. 4. We detail the analysis method we have followed in order to try to improve the universal relations in Appendix A.

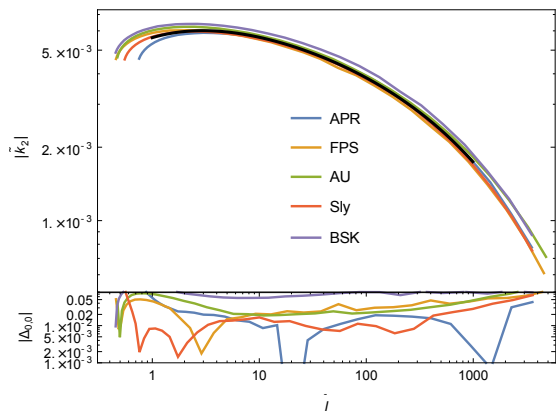


FIG. 3. The dispersion for the gravitomagnetic Love number of irrotational stars. The black thick curve is the best fit with parameters given in Table II.

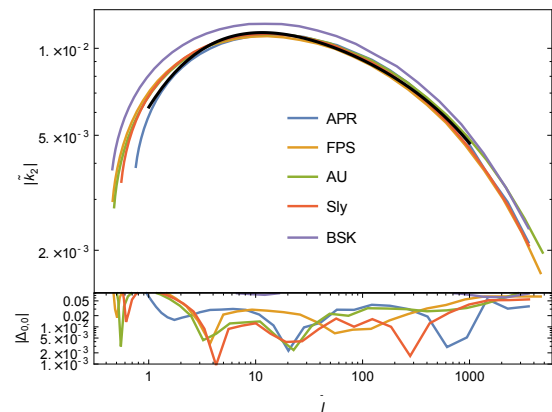


FIG. 4. The dispersion for the gravitomagnetic Love number of stars in strict hydrostatic equilibrium. The black thick curve is the best fit with parameters given in Table I.

IV. DISCUSSION AND CONCLUSION

In this short paper, we have investigate the universality of the gravimagnetic Love number of irrotational stars and stars in strict equilibrium, by looking for correlations between the latter and the dimensionless moment of inertia, defined by $\bar{I} = I/M_s^3$. We have used 5 different tabulated equations of state, namely APR, SLy, AU, BSK and FPS, that are commonly used in the literature on universal relations.

We found that both Love numbers normalized follow approximate universal relation, with a dispersion less than $\approx 10\%$ for the strict hydrostatic equilibrium stars and less than $\approx 5\%$ for the irrotational stars. Note that previous results were reported on the universal behaviour of the strict hydrostatic equilibrium case in [25]. We found that modifying slightly the normalisation of the gravitomagnetic Love number and of the dimensionless moment of inertia according to $C^a \bar{I}$ and $C^b \bar{k}_2$, it is pos-

sible to reduce the maximal dispersion by a factor almost 2 in the case of strict equilibrium. However, the average standard deviation did not change significantly. We gave the best values of a, b that were determined by minimising the total standard deviation, obtained by computing the deviation to the average curve of all the data we have generated. We stress that the deviation and mean deviation we computed take into account lighter configurations, where it is expected that the universality holds less.

We expect this universal relation to extend to a the rotation induced quadrupole, as well as to higher gravitomagnetic irrotational multipoles, along with [25]. Finally, we note that there is a slight improvement in the universality when taking into account the internal motion of the stars, induced by the gravitomagnetic tidal field. We observed that the mean (resp. max) standard deviation in our data is 4% (resp. 14.5%) for the irrotational fluid, while it is 7% (resp. 33%) in the case of strict static equilibrium. We expect this feature to hold for higher multipoles, strengthening the correlations found in [25].

V. ACKNOWLEDGEMENT

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Appendix A: Choosing the normalisation

Following [34], we investigated a deformation of the universal relation by looking for correlations between $\bar{I}C^a$ and \tilde{k}_2C^b , for some a, b to be determined.

We adopted the following strategy: first we generated tables of $(\bar{I}C^a, \tilde{k}_2C^b)$ for all the equations of state we built for various neutron star configurations. Let us call these data $(I_a^K, K_b^K)_i$ and their interpolating function $\mathcal{I}_{ab}^K(k)$, where K labels the equations of state. Next we have computed the average of the interpolation of these data, and obtained a function $\langle \mathcal{I}_{ab}(k) \rangle$:

$$\langle \mathcal{I}_{ab}(k) \rangle = \frac{1}{N} \sum_{K=1}^N \mathcal{I}_{ab}^K(k), \quad (\text{A1})$$

where N is the total number of equations of state.

Then the dispersion of the data of the equation of state number K with respect to the average values is given by

$$\Delta_{a,b}^K = \left| \frac{I_a^K - \langle \mathcal{I}_{ab}(K_b^K) \rangle}{\langle \mathcal{I}_{ab}(K_b^K) \rangle} \right|. \quad (\text{A2})$$

It should be stressed that the sampling values of K_b^K depend on K . Thus it is not a good idea to use directly these datas as they are. Instead, we resampled the data on an log-equally spaced sample in the common range of

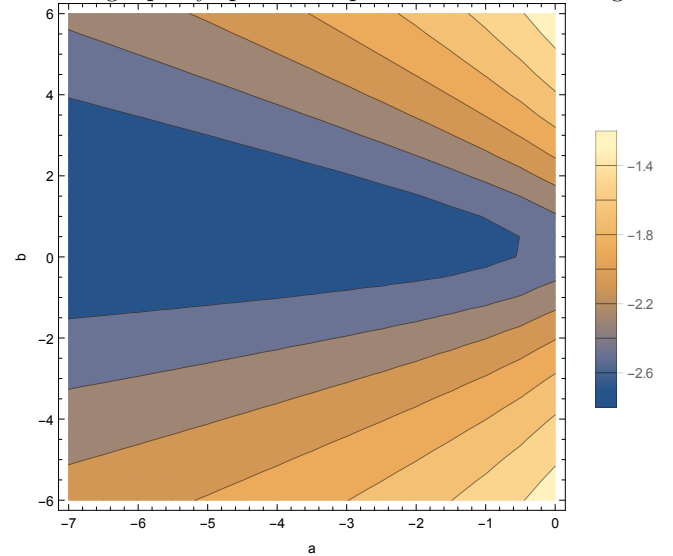


FIG. 5. \log_{10} of the cumulative standard deviation of the relative difference between the curves in the $(j_l C^a, \bar{I}C^b)$ plane and their average, computed with 5 different equations of state for irrotational stars.

the K_a^K datas. This resampling is now independent of the equation of state. Let us call the dispersions computed on the resampled data D_{ab}^K . We then compute the variance of the joint data, defining

$$D_{ab} = \text{Var}(\cup_{K=1}^N D_{ab}^K), \quad (\text{A3})$$

where Var is the variance.

Fig. 5 shows $\sqrt{D_{ab}}$ as a function of a and b . Of course, this plot depends on the specific choice of the equations of state set, but we expect the minimal region to remain roughly the same. As a sanity check, we checked that the mean value of the dispersion is a small number, typically find of the order of $\approx 10^{-3}$.

Appendix B: Coefficients of the fit

a	b	d_0	d_1	d_2	d_3	d_4	$\langle \Delta \rangle$	$\text{Max} \Delta $
0	0	$-5.07 + 0$	$5.96 - 1$	$-1.99 - 1$	$2.58 - 2$	$-1.51 - 3$	0.070	0.330
-5	0	$-8.60 + 0$	$8.69 - 1$	$-6.53 - 2$	$2.12 - 3$	$-2.79 - 5$	0.050	0.193

TABLE I. Values of the parameter of (11), for the direct relation between \bar{I} and σ_2 and for a different choice of normalisation. This case is the case considered in previous literature and referred to as strict hydrostatic equilibrium in [9]. $\langle |\Delta| \rangle$ is the average absolute deviation over all the axis, and $\text{Max}|\Delta|$ is largest deviation we observe over all the axis. The numbers are presented as $X \pm Y = X \cdot 10^{\pm Y}$.

a	b	d_0	d_1	d_2	d_3	d_4	$\langle \Delta \rangle$	$\text{Max} \Delta $
0	0	$-5.18 + 0$	$1.29 - 1$	$-7.16 - 2$	$8.58 - 3$	$-6.47 - 4$	0.040	0.144
-3	0	$-5.86 + 0$	$2.80 - 1$	$-3.38 - 2$	$1.48 - 3$	$-3.24 - 5$	0.034	0.131

TABLE II. Values of the parameter of (11), for the direct relation between \bar{I} and \tilde{k}_2 and for a different choice of normalisation. This is the irrotational fluid case considered in [9], where internal motion is present. $\langle |\Delta| \rangle$ is the average absolute deviation over all the axis, and $\text{Max}|\Delta|$ is largest deviation we observe over all the axis. The numbers are presented as $X \pm Y = X \cdot 10^{\pm Y}$.

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- [1] A. E. H. Love, *Some Problems of Geodynamics*. Cambridge University Press, Cambridge, England, 1911.
- [2] T. Damour and A. Nagar, “Relativistic tidal properties of neutron stars,” *Phys. Rev. D* **80** (2009) 084035, [arXiv:0906.0096 \[gr-qc\]](#).
- [3] T. Binnington and E. Poisson, “Relativistic theory of tidal Love numbers,” *Phys. Rev. D* **80** (2009) 084018, [arXiv:0906.1366 \[gr-qc\]](#).
- [4] P. Pani, L. Gualtieri, A. Maselli, and V. Ferrari, “Tidal deformations of a spinning compact object,” [arXiv:1503.07365 \[gr-qc\]](#).
- [5] P. Landry and E. Poisson, “Tidal deformation of a slowly rotating material body. I. External metric,” [arXiv:1503.07366 \[gr-qc\]](#).
- [6] S. Chakrabarti, T. Delsate, and J. Steinhoff, “Effective action and linear response of compact objects in Newtonian gravity,” *Phys. Rev. D* **88** (2013) 084038, [arXiv:1306.5820 \[gr-qc\]](#).
- [7] S. Chakrabarti, T. Delsate, and J. Steinhoff, “New perspectives on neutron star and black hole spectroscopy and dynamic tides,” [arXiv:1304.2228 \[gr-qc\]](#).
- [8] M. Favata, “Are neutron stars crushed? gravitomagnetic tidal fields as a mechanism for binary-induced collapse,” *Phys. Rev. D* **73** (2006) 104005, [arXiv:astro-ph/0510668 \[astro-ph\]](#).
- [9] P. Landry and E. Poisson, “Gravitomagnetic response of an irrotational body to an applied tidal field,” [arXiv:gr-qc/1504.0660](#).
- [10] K. Yagi and N. Yunes, “I-Love-Q: Unexpected universal relations for neutron stars and quark stars,” *Science* **341** (2013) 365–368, [arXiv:1302.4499 \[gr-qc\]](#).
- [11] K. Yagi, K. Kyutoku, G. Pappas, N. Yunes, and T. A. Apostolatos, “Effective No-Hair Relations for Neutron Stars and Quark Stars: Relativistic Results,” *Phys. Rev. D* **89** (2014) no. 12, 124013, [arXiv:1403.6243 \[gr-qc\]](#).
- [12] G. Pappas and T. A. Apostolatos, “Universal behavior of rotating neutron stars in GR: Even simpler than their Newtonian counterparts,” [arXiv:1311.5508 \[gr-qc\]](#).
- [13] K. Chatziioannou, K. Yagi, and N. Yunes, “Toward realistic and practical no-hair relations for neutron stars in the nonrelativistic limit,” *Phys. Rev. D* **90** (2014) no. 6, 064030, [arXiv:1406.7135 \[gr-qc\]](#).
- [14] D. D. Doneva, S. S. Yazadjiev, N. Stergioulas, and K. D. Kokkotas, “Breakdown of I-Love-Q universality in rapidly rotating relativistic stars,” *Astrophys. J. Lett.* **781** (2014) L6, [arXiv:1310.7436 \[gr-qc\]](#).
- [15] S. Chakrabarti, T. Delsate, N. Grlebeck, and J. Steinhoff, “The I-Q relation for rapidly rotating neutron stars,” *Phys. Rev. Lett.* **112** (2014) 201102, [arXiv:1311.6509 \[gr-qc\]](#).
- [16] A. Maselli, V. Cardoso, V. Ferrari, L. Gualtieri, and P. Pani, “Equation-of-state-independent relations in neutron stars,” *Phys. Rev. D* **88** (2013) 023007, [arXiv:1304.2052 \[gr-qc\]](#).
- [17] B. Haskell, R. Ciolfi, F. Pannarale, and L. Rezzolla, “On the universality of I-Love-Q relations in magnetized neutron stars,” [arXiv:1309.3885 \[astro-ph.SR\]](#).
- [18] D. D. Doneva, S. S. Yazadjiev, K. V. Staykov, and K. D. Kokkotas, “Universal I-Q relations for rapidly rotating neutron and strange stars in scalar-tensor theories,” *Phys. Rev. D* **90** (2014) no. 10, 104021,

- arXiv:1408.1641 [gr-qc].
- [19] P. Pani and E. Berti, “I-Love-Q, Spontaneously: Slowly Rotating Neutron Stars in Scalar-Tensor Theories,” arXiv:1405.4547 [gr-qc].
 - [20] B. Kleihaus, J. Kunz, and S. Mojica, “Quadrupole Moments of Rapidly Rotating Compact Objects in Dilatonic Einstein-Gauss-Bonnet Theory,” *Phys.Rev.* **D90** (2014) no. 6, 061501, arXiv:1407.6884 [gr-qc].
 - [21] Y. Sham, L. Lin, and P. Leung, “Testing universal relations of neutron stars with a nonlinear matter-gravity coupling theory,” *Astrophys.J.* **781** (2014) no. 2, 66, arXiv:1312.1011 [gr-qc].
 - [22] S. Alexander and N. Yunes, “Chern-Simons Modified General Relativity,” *Phys.Rept.* **480** (2009) 1–55, arXiv:0907.2562 [hep-th].
 - [23] T. Delsate, D. Hilditch, and H. Witek, “The Initial Value Formulation of Dynamical Chern-Simons Gravity,” *Phys. Rev. D* (2014) , arXiv:1407.6727 [gr-qc].
 - [24] M. Agathos, J. Meidam, W. Del Pozzo, T. G. F. Li, M. Tompitak, *et al.*, “Constraining the neutron star equation of state with gravitational wave signals from coalescing binary neutron stars,” arXiv:1503.05405 [gr-qc].
 - [25] K. Yagi, “Multipole Love Relations,” *Phys.Rev.* **D89** (2014) no. 4, 043011, arXiv:1311.0872 [gr-qc].
 - [26] J. B. Hartle, “Slowly rotating relativistic stars. 1. Equations of structure,” *Astrophys.J.* **150** (1967) 1005–1029.
 - [27] Z. Andrade and R. H. Price, “Excitation of the odd parity quasinormal modes of compact objects,” *Phys.Rev.* **D60** (1999) 104037, arXiv:gr-qc/9902062 [gr-qc].
 - [28] T. Damour and A. Nagar, “Effective one body description of tidal effects in inspiralling compact binaries,” *Phys. Rev. D* **81** (2010) 084016, arXiv:0911.5041 [gr-qc].
 - [29] A. Akmal, V. Pandharipande, and D. Ravenhall, “The Equation of state of nucleon matter and neutron star structure,” *Phys.Rev.* **C58** (1998) 1804–1828, arXiv:nuc1-th/9804027 [nucl-th].
 - [30] S. Goriely, N. Chamel, and J. Pearson, “Skyrme-Hartree-Fock-Bogoliubov nuclear mass formulas: Crossing the 0.6 MeV threshold with microscopically deduced pairing,” *Phys.Rev.Lett.* **102** (2009) 152503, arXiv:0906.2607 [nucl-th].
 - [31] R. B. Wiringa, V. Fiks, and A. Fabrocini, “Equation of state for dense nucleon matter,” *Phys.Rev.* **C38** (1988) 1010–1037.
 - [32] F. Douchin and P. Haensel, “A unified equation of state of dense matter and neutron star structure,” *Astron.Astrophys.* **380** (2001) 151, arXiv:astro-ph/0111092 [astro-ph].
 - [33] C. Lorenz, D. Ravenhall, and C. Pethick, “Neutron star crusts,” *Phys.Rev.Lett.* **70** (1993) 379–382.
 - [34] B. Majumder, K. Yagi, and N. Yunes, “Improved Universality in the Neutron Star Three-Hair Relations,” arXiv:1504.02506 [gr-qc].